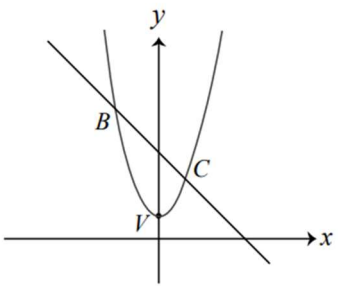
Name_____ **Date:**_____

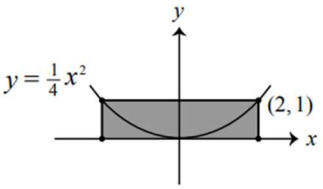
Math 10/11 Honors Ch2 Review #5 Quadratic Functions and Discriminant Review

1	Find the roots of the following: i. $x^2 = -5x - 6.$ ii. $x^2 - 3x - 40 = 0.$ iii. $2x^2 + \frac{1}{3}x - \frac{2}{3} = 0.$
2	Convert the following quadratic equations to APQ form and then find the coordinates of the vertex, equation of the axis of symmetry, domain and range. i) $y = x^2 + 6x - 5$ ii) $y = 2x^2 - 8x + 11$ iii) $y = \frac{2}{3}x^2 + 6x - 11$
3	Find the sum of the solutions of the equation $8^{x^2+3x+10} = 4^{x^2-x}$. (MAΘ 1992)
4	Find all real values of x which satisfy $\sqrt{x^2+1} + x^2 + 1 = 90$. (MAΘ 1991)
5	Find all real n such that $1 + 2^n + 2^{2n} = 73$. (MAΘ 1987)
6	The function y = -16t^2 + 9t + 248 models the height y in feet of a stone t seconds after it is dropped from the edge of a vertical cliff. How long will it take the stone to hit the ground ? Round to the nearest hundredth of a second. Find the domain and range of the function:
7	Suppose that $a > \frac{1}{2}$ and that the parabola with equation $y = ax^2 + 2$ has vertex V. The parabola intersects the line with equation $y = -x + 4a$ at points B and C, as shown. If the area of $\triangle VBC$ is $\frac{72}{5}$, determine the value of a. 

8	A manufacturer determines that the number of drills it can sell is given by the formula $D = -4p^2 + 160p - 305$, where p is the price of the drills in dollars. a. At what price will the manufacturer sell the maximum number of drills? b. What is the maximum number of drills that can be sold?
9	A bicycle shop sells 55 bikes a week at a cost of \$1800 per bike. For every \$50 decrease in the price of the bike, sales will go up by 2 bikes per week. a) Write the Price of the bike as a function of the Quantity (Number of bikes sold in a week) b) What price per bike will yield the maximum revenue in a week? c) What is the maximum revenue? d) Suppose that the function for Cost is: $C = 400x + 850$ per week, where "x" is the number of bikes sold in a week, what does "x" need to be to make a profit greater than zero? e) What quantity in bikes sold will generate the maximum profit? f) What range in price will the profit be at least \$75,000 per week?
10	There are functions $f(x)$ with the following properties: • $f(x) = ax^2 + bx + c$ for some integers a, b and c with $a > 0$, and • $f(p) = f(q) = 17$ and $f(p+q) = 47$ for some prime numbers p and q with $p < q$. For each such function, the value of $f(pq)$ is calculated. The sum of all possible values of $f(pq)$ is S. What are the rightmost two digits of S?
11	Suppose that $f(x) = x^3 - px^2 + qx$ and $g(x) = 3x^2 - 2px + q$ for some positive integers p and q. (a) If $p = 33$ and $q = 216$, show that the equation $f(x) = 0$ has three distinct integer solutions and the equation $g(x) = 0$ has two distinct integer solutions. (b) Suppose that the equation $f(x) = 0$ has three distinct integer solutions and the equation $g(x) = 0$ has two distinct integer solutions. Prove that (i) p must be a multiple of 3, (ii) q must be a multiple of 9, (iii) $p^2 - 3q$ must be a positive perfect square, and (iv) $p^2 - 4q$ must be a positive perfect square.

12

The parabola with equation $y = \frac{1}{4}x^2$ has its vertex at the origin and the y-axis as its axis of symmetry. For any point (p, q) on the parabola (not at the origin), we can form a *parabolic rectangle*. This rectangle will have one vertex at (p, q), a second vertex on the parabola, and the other two vertices on the x-axis. A parabolic rectangle with area 4 is shown.



(a) A parabolic rectangle has one vertex at $(6, 9)$. What are the coordinates of the other three vertices?

(b) What is the area of the parabolic rectangle having one vertex at $(-3, 0)$?

(c) Determine the areas of the two parabolic rectangles that have a side length of 36.

(d) Determine the area of the parabolic rectangle whose length and width are equal.

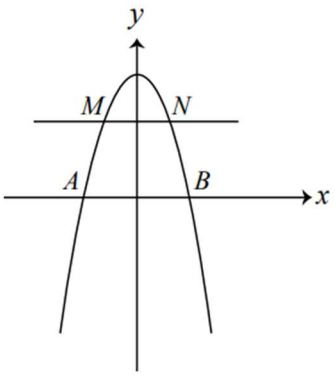
13

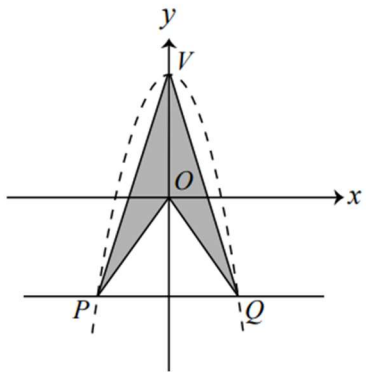
The parabola with equation $y = -x^2 + 16$ intersects the x-axis at points A and B and the horizontal line with equation $y = 7$ at points M and N, as shown.

(a) Determine the coordinates of A and B.

(b) Determine the area of the trapezoid $MNBA$.

(c) Suppose that O is the origin and V is the vertex of the parabola. The line $y = -33$ intersects the parabola at points P and Q. Determine the area of quadrilateral $VPOQ$, which is shaded in the diagram below.





14

Problem 12. The integer lengths of two legs of a right triangle are the two roots of the quadratic equation $x^2 - (m+2)x + 4m = 0$ about x. What is the sum of all possible values of m?

15

Problem 13. The equation $x^2 + mx - m + 1 = 0$ has two distinct positive integer roots. What is the value of m?

16	**Problem 14.** (2002 China Middle School Math Competition) Find all possible values of rational number r such that the following equation always has the integer roots: $rx^2+(r+2)x+3r-2=0$.
17	**Problem 15.** (China JiangSu Middle School Math Competition) Find all possible values of real number r such that the following equation always has the two integer roots: $rx^2+(r+1)x+r-1=0$.
18	**Problem 16** Find all possible values of real positive number a such that the following equation always has the two integer roots: $x^2-ax+4a=0$.
19	**Problem 1.** Find the smallest integral value of k such that $5x(kx-6)-x^2+9=0$ has no real roosts.
20	Consider the quadratic equation $x^2-(r+7)x+r+87=0$ where r is a real number. This equation has two distinct real solutions x which are both negative exactly when $p<r<q$, for some real numbers p and q. The value of p^2+q^2 is **(A)** 7618 **(B)** 698 **(C)** 1738 **(D)** 7508 **(E)** 8098